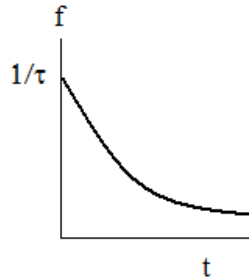


Homework 3 Solutions

Ch. 5 6,12,20,22,34,36

5.6

(a)



$$(b) \quad \int_0^{\infty} \frac{1}{\tau} e^{-t/\tau} dt = \int_0^{\infty} e^{-z} dz = 1$$

$$(c) \quad \bar{t} = \int_0^{\infty} t f(t) dt = \int_0^{\infty} \frac{t}{\tau} e^{-t/\tau} dt = \tau \int_0^{\infty} z e^{-z} dz = \tau$$

For parts (b) and (c), we used the substitution $z = t/\tau$ and the identity $n! = \int_0^{\infty} z^n e^{-z} dz$

5.12

Since we are looking at the width (not the position) of the distribution, we can consider a Gaussian centered about the origin. Then, the Gaussian has $1/2$ its maximum value when

$$\exp\left(-\frac{x^2}{2\sigma^2}\right) = \frac{1}{2} \quad \frac{x^2}{2\sigma^2} = \ln 2 \quad x = \sigma\sqrt{2\ln 2}$$

Therefore, $FWHM = 2x = 2\sigma\sqrt{2\ln 2}$

5.20

- This is a range between -1 and $+1 \sigma$. Therefore, 68.3% of the population would fall in this range, or 683 men.
- This is over $+1 \sigma$. Therefore, $0.5 - 0.3413 = 15.9\%$ fall in this range, or 159 men.
- This is over $+3 \sigma$. $0.5 - 49.87 = 0.13\%$ fall in this range, or about 1 man.
- This is between -2 and -1σ . 15.9% are below -1σ , while 2.3% are below -2σ , so 13.6% fall in this intermediate range, or about 136 men.

5.22

First, move the Gaussian to the origin by setting X to zero. This corresponds to the substitution $x \rightarrow x + X$. Then,

$$P(t\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-t\sigma}^{+t\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right) dx$$

Now, let $x = \sigma z$. Then, $dx = \sigma dz$, and z runs between $-t$ and $+t$.

$$P(t) = \frac{1}{\sqrt{2\pi}} \int_{-t}^{+t} \exp\left(-\frac{z^2}{2}\right) dz = \text{erf}(t)$$

5.34

Utilizing the table of the erf function in Appendix A of the book, we find that 5% significance requires $t > 1.96$, 2% significance requires $t > 2.32$, and 1% significance requires $t > 2.58$.

5.36

- (a) The discrepancy is $(15-13) \pm \sqrt{1^2 + 1^2} = 2.0 \pm 1.4$
- (b) In this case, $t = 2/1.4 \approx 1.4$, which means that there is a 16% chance of getting results with this discrepancy by random chance alone. Thus, this discrepancy is not significant at the 5% level; 5% confidence would require the probability of a false positive due to random chance to be below 5%, and thus require a t value of at least 1.96.